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SIMULATING A 6D COOLING CHANNEL IN BDSIM

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Abstract

Muon colliders offer high-luminosity, multi-TeV collisions without significant synchrotron radiation but require further exploration of muon production, acceleration, cooling, and storage techniques. A proposed 6D cooling demonstrator aims to extend the MICE experiment's validation of transverse ionization cooling to also reduce longitudinal emittance, using bunched muon beams and incorporating RF cavities for reacceleration. The cooling lattice includes solenoids for tight focusing, dipoles for beam dispersion, and wedge absorbers for differential energy loss. This paper presents a complete implementation of cooling channels for BDSIM, a Geant4-based accelerator simulation tool, using appropriate analytic field models to account for fringe-field-dominated magnets. Components have been tested individually and validated against other tracking codes such as G4BeamLine. A tracking study leveraging this implementation is presented, simulating and optimizing a rectilinear cooling channel for the 6D cooling demonstrator. The analysis incorporates beam parameters from existing proton drivers, using outputs from targetry and capture system designs.

INTRODUCTION

The muon collider is a promising candidate for a multi-TeV lepton collider, with the potential to reach centre-of-mass energies of up to 10 TeV at high luminosity. Realising such a machine requires beams with high brightness, and therefore low transverse and longitudinal emittance. In a proton driver scheme, muons are produced as tertiary particles from pion decays, resulting in beams with large momentum and spatial spreads. Consequently, substantial beam cooling is necessary. As conventional techniques such as synchrotron radiation or stochastic cooling are ineffective for muons due to their short lifetime, a novel approach known as ionisation cooling has been proposed.

Ionisation cooling involves passing the beam through an absorber, where muons lose momentum via ionisation energy loss in all directions. Longitudinal momentum is then restored using RF cavities, reducing transverse emittance. In rectilinear (6D) cooling channels, dipole magnets introduce dispersion, correlating particle momentum with transverse position. A wedge-shaped absorber then causes higher-momentum particles to lose more energy, reducing energy spread at the cost of increased transverse emittance—effectively transferring emittance from the longitudinal to the transverse phase space and achieving net 6D cooling. In contrast, final (4D) cooling channels target only transverse emittance and use cylindrical absorbers and no dipoles. Multiple Coulomb scattering can degrade performance and is mitigated by strong solenoidal focusing at the absorber and by using low-Z materials such as liquid hydrogen or lithium hydride to minimise scattering while maintaining energy loss.

6D Cooling Demonstrator

The principle of ionisation cooling was demonstrated by the MICE experiment. [1] Building on this, a 6D cooling demonstrator is proposed to establish proof of principle for six-dimensional emittance reduction. The demonstrator would cool a bunched muon beam, necessitating studies into its production, capture, instrumentation. Upstream, a beam preparation section is composed of collimators and RF cavities in order to perform a phase rotation and to impose a bunch structure. The cooling channel itself requires a compact lattice of RF cavities, solenoids, dipoles, and absorbers, along with instrumentation along and after the cooling channel for beam monitoring and emittance measurement.

BDSIM

The Beam Delivery Simulation (BDSIM) software, developed in Europe for modelling accelerator lattices that include beam-intersecting devices, has been employed in studies of next-generation collider candidates such as FCC-hh, CLIC, and ILC. [2]. While ionisation cooling simulations have traditionally relied on G4BeamLine, its lack of recent updates has motivated the search for a more actively maintained and flexible alternative. BDSIM has emerged as a strong candidate for muon cooling studies, providing a modern, Geant4-based framework that benefits from ongoing development and community support. The work presented here details the implementation and integration of the physical models required to simulate all lattice components essential for six-dimensional ionisation cooling, as well as various model extensions and code optimisations. A full report, along with validations has been presented in [3].

IMPLEMENTATION

The underlying code has been developed in `C++` within the BDSIM framework. It has been made publicly available

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on GitHub and pulled into the main BDSIM codebase. The code is released under the GNU General Public License Version 3.

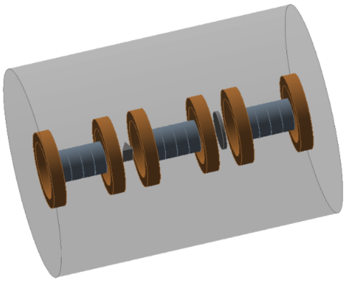

Figure 1: A 3D rendering of a cooling channel simulated using BDSIM.

A new BDSIM element, `muoncooler`, has been implemented to enable the simulation of a complete 6D or 4D muon cooling system. An example of a cooling channel consisting of three cooling cells and two types of absorbers is shown in Fig. 1. This implementation allows for the construction of an entire cooling lattice within a single element. The electromagnetic fields from each component are superimposed, producing a continuous six-component field vector throughout the channel, ensuring that fringe field effects from all magnetic elements are accurately included along the full extent of the lattice.

Dipole Model

Two dipole models have been written for a cooling channel. A hard edge dipole field, similar to other simulation codes has been implemented. However, the dipole magnets in the rectilinear cooling lattice feature a large aperture relative to their on-axis length, resulting in a magnetic field that is dominated by fringe effects. To accurately model these effects, a dipole field model has been implemented based on the analytical treatment described by Muratori et al. [4], in which the fringe fields are represented using Enge functions. The magnetic field for each magnet is modelled as the sum of an entry (left), B_l and exit (right) B_r. These fringe field enge functions are stitched together as

$$B_y = \zeta B_0 (B_{y,l} + B_{y,r} - 1), \tag{1}$$

$$B_z = \zeta B_0 (B_{z,l} + B_{z,r}). \tag{2}$$

where B_0 is the nominal field and ζ is a normalisation factor to ensure the fields converge to B_0 inside the magnet. An example of this is shown in Fig. 2.These were compared to both analytic and G4Beamline field maps, showing good agreement.

Solenoid Model

Two models have been implemented to estimate the magnetic field produced by a solenoid coil. The analytic expressions for the cylindrical magnetic field components follow the treatment outlined by Derby et al. [5]. In the simplest case, the solenoid is modelled as a single current-carrying sheet. For a more realistic three-dimensional representation, multiple such sheets can be layered at increasing radii to approximate a cylindrical block, thereby capturing the spatial extent and field distribution of a physical solenoid more accurately. The implementation also includes the option to set user defined tolerances to implement bounding boxes, so that complicated elliptic integrals do not have to be computed at all points for all solenoids. The solenoid models were also compared against G4BL field maps, along with a like for like tracking study through a single solenoid, showing good agreement.

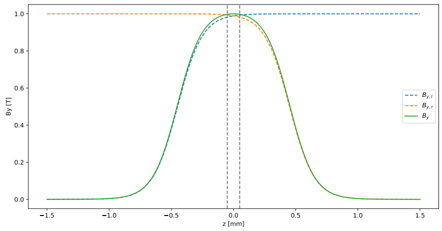

Figure 2: Example B_y dipole field on axis ($x = 0$, $y = 0$).

RF Cavity

The RF cavities are modelled as pillboxes operating in the TM_{010} mode. The electromagnetic field within each cavity is parameterised by the peak electric field E, frequency f, phase ψ, and a global time offset T_0. In this mode, the field is characterised entirely by a longitudinal electric component (E_z) and an azimuthal magnetic component (B_ϕ), given by:

$$E_z(r_n, z, t) = E J_0(r_n) \cos\left(2\pi f (t - T_0) + \psi\right), \tag{3}$$

$$B_\phi(r_n, z, t) = \mu_0 \frac{E}{Z_0} J_1(r_n) \sin\left(2\pi f (t - T_0) + \psi\right), \tag{4}$$

where J_0 and J_1 are Bessel functions of the first kind, and r_n is the radial distance normalised to the first zero of J_0. Z_0 denotes the impedance of free space, and the magnetic field expression assumes a vacuum, using the vacuum permeability μ_0 to relate B to H. This is the standard pre existing implementation of BDSIM, and has been seperately verified.

Absorbers

Models for particle–matter interactions are inherited from, and handled by, the underlying GEANT4 framework. In addition to these built-in capabilities, custom absorber geometries have been implemented within the simulation, including both wedge-shaped and cylindrical absorbers. These geometries are essential for accurately modelling emittance exchange and energy loss processes in muon cooling channels. These also showed good agreement with other frameworks such as ICOOL and G4BL.

BEAM TRACKING AND RESULTS

Following the implementation and validation of each component of a muon cooling cell, a full rectilinear cooling lattice was simulated. The lattice is based on the demonstrator cooling cell presented in [6]. A detailed description of the cooling cell parameters can be found in Table 2 of that reference. The tracking study sequentially adds each type of element to the full muon cooling lattice, and the results of each step have been shown below.

First, a pure solenoid lattice was instantiated. The optical beta function along the cooling cell for a 200 MeV/c beam is shown in Fig. 3. Minima are observed at the extremities of the cell—where the absorbers are located—indicating tight beam focusing at these points.

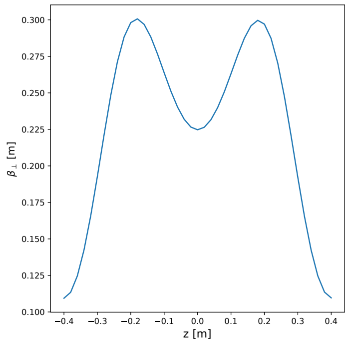

Figure 3: Optical beta function over the extent of the cooling cell for a 200 MeV/c beam.

Dipole magnets were introduced to generate dispersion. The closed-orbit trajectories in the x and y planes, tracked across two cooling cells for three different momenta, are shown in Fig. 4. As intended, significant dispersion is evident at the absorber locations, particularly in the x plane.

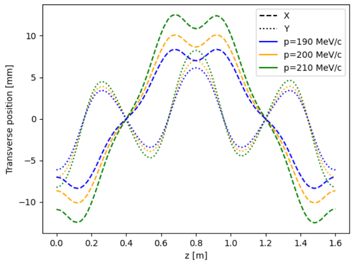

Figure 4: x and y closed orbit trajectories at three different momenta.

The full cooling lattice was completed by adding 704 MHz RF cavities and lithium hydride wedge absorbers (20 mm thick on axis). A full beam was tracked through the resulting ~80 m-long cooling channel. The input beam was a multivariate Gaussian distribution with an initial transverse emittance of approximately 2.5 mm and a longitudinal emittance of roughly 1.8 eV·s. The emittance evolution along the lattice is shown in Fig. 5, where both transverse and longitudinal cooling are observed.

An initial mismatch in the longitudinal phase space causes early emittance growth, leading to particle losses in the beam tails. This mismatch also induces oscillations in the beam momentum, which manifest as oscillatory behaviour in the longitudinal emittance. These effects could be mitigated by preparing a properly matched input beam.

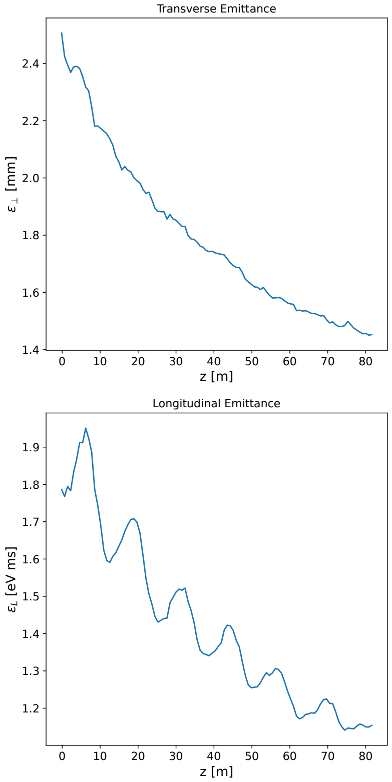

Figure 5: The evolutions of the (left) transverse and (right) longitudinal emittances across a ~ 80 m lattice.

CONCLUSION

A dedicated `muoncooler` beamline element has been implemented in BDSIM. The individual components have been validated against analytical models and existing simulation tools such as G4Beamline. Demonstrations of both four-dimensional (4D) and six-dimensional (6D) ionisation cooling have been successfully performed. This development establishes BDSIM as a viable framework for simulating the full ionisation cooling complex required for a muon collider.

Future work will focus on benchmarking 6D cooling performance against G4Beamline and, potentially, RF-Track. Further planned optimisations also aim to improve the efficiency and realism of the code.

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