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DESIGN OF BEAM PHASE SPACE DISTRIBUTION TO REALIZE PRECISE THREE-DIMENSIONAL BEAM INJECTION AT J-PARC MUON g-2/EDM EXPERIMENT

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Abstract

In the J-PARC muon g-2/EDM experiment, a three-dimensional beam injection will be adopted to inject a low-emittance 300 MeV/c muon beam into a storage region with a compact radius where the magnetic field is precisely tuned for a muon g-2 measurement.

The method to design the injecting beam distribution was previously unclear, as muons pass through an area with a largely position-dependent, non-linear, and time-dependent magnetic field during the injection process. This paper presents a method to evaluate an acceptance on the injecting beam phase space.This enables us to search for beam distributions fitting the acceptance and to maximize the number of muons injected into the storage region.

The evaluated acceptance shows strong XY coupling and narrow width as a feature of three-dimensional injection, which reveals a precise beam control required to realize this injection scheme.

THREE-DIMENSIONAL PRECISE BEAM INJECTION

Muon g-2/EDM Experiment at J-PARC

Anomalous magnetic moment (g-2) of a muon is a good probe to search for new physics beyond the Standard Model because it can be experimentally measured and theoretically predicted with high precision. Measurements held at BNL and FNAL [1] showed a large discrepancy from the theoretical prediction from the dispersive approach, while recent prediction utilizing lattice QCD gives consistent value with the measurement [2, 3].

To validate the discrepancy from the experimental side, a new measurement of muon g-2 is planned at J-PARC [4]. The J-PARC experiment is based on an independent concept from the previous experiments. It uses low emittance muon beam obtained by a reacceleration of thermal muons from a laser ionization of thermal muoniums. This allows us to measure muon g-2 without an electric field in the compact storage orbit with more uniform magnetic field.

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Three-dimensional Beam Injection

To realize the J-PARC experiment, we need to inject a low emittance muon beam ($P = 300\,\mathrm{MeV/c}$, $\epsilon^{rms}_{unnormalized} = 0.1\,()\mathrm{mm-mrad}$) into a compact storage orbit ($R = 33\,\mathrm{cm}$) in a solenoid magnet ($B_z = 3.0\,\mathrm{T}$), and store it inside a storage region ($|z| < 5\,\mathrm{cm}$) during its physics measurement. Because of a small cyclotron period of 7 ns, usual two-dimensional beam injection is not possible as we cannot apply sufficient kick within a cyclotron period. To solve this, a new three-dimensional beam injection scheme was proposed [5] and being developed.

Figure 1 shows a concept of this scheme. We utilize two coordinate system in this paper. One is the cylindrical coordinate (r, θ, z) on the solenoid magnet, where z is the vertical axis, and $z = 0$ corresponds to the storage plane. The other is the beam coordinate (X, X', Y, Y', Z, Z') defined on the reference orbit, where X axis is horizontal.

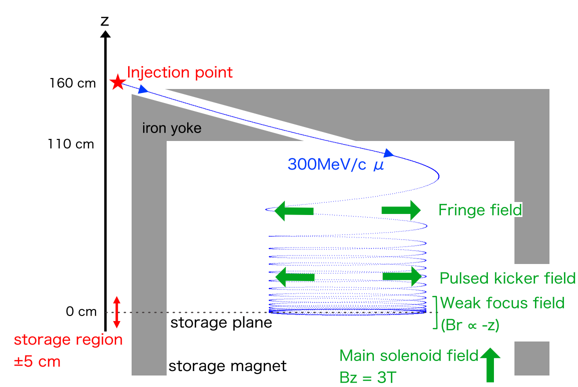

Figure 1: Concept of three-dimensional beam injection.

The muon beam is injected from the top of the solenoid magnet with a pitch angle passing through a tunnel in the iron yoke of the magnet. The beam follows a spiral orbit inside the magnet. A pulsed radial magnetic field is applied to control the vertical component of muon momentum by a kicker coil and power supply (peak current: about 1 kA) to guide muons onto the designed horizontal storage plane where weak focusing magnetic field is applied. This injection process takes about 130 ns, and we can utilize this time period to kick muons. Stored muons follow a cyclotron motion with a period of 7 ns, and a vertical simple harmonic oscillation

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by the weak focusing field ($Br = -n\frac{z}{R}Bz, n = 1.5 \times 10^{-4}$), called as a vertical betatron oscillation (VBO), with a period of 600 ns.

In the J-PARC muon g-2/EDM experiment, almost all muons must be stored within a beam storage region defined as $|z| < 5$ cm around the storage plane. The magnetic field in this region is precisely measured and tuned to reduce systematic uncertainties of the g-2 measurement. Therefore, VBO amplitude of stored muons must be minimized, as it defines the vertical distribution of them.

To minimize VBO amplitude of stored muons, we can select an orbit stored with zero amplitude as a reference orbit and a central orbit. In addition to that, phase space distribution of the injected beam also needs to be correctly designed because the VBO amplitude distribution is determined from the vertical phase space distribution when the pulsed kicker field is finished which results from the phase space distribution at the injection point. An upstream beamline is designed to control the beam distribution at the injection point before entering the solenoid magnet [1] including its XY coupling.

BEAM DISTRIBUTION FOR THREE-DIMENSIONAL INJECTION

As discussed above, beam phase space distribution at the injection point must be correctly designed to inject almost all muons into the storage region. On the other hand, a method to design this was previously unclear. This paper defines an acceptance at the injection point as a subspace on the beam phase space corresponding to the storage region after injection. In other words, being inside the acceptance at the injection point is a necessary and sufficient condition for each muon to be injected into the storage region. Once we evaluate the acceptance, we can design our beam distribution to match it as much as possible.

Evaluation Method of Acceptance

The acceptance at the injection point is defined as a subspace on the beam phase space ($R^4|X, X', Y, Y'$) corresponding to the storage region after injection. The relation between two phase space is determined from particle motion under the given field, and the acceptance should be able to be evaluated. However, in the three-dimensional injection, this relation is not trivial because of a complicated magnetic field. The magnetic field felt by muons during injection has many components: main solenoid field, its fringe field, kicker field, and weak focusing field. It is also affected by the iron yoke and tunnel structure near the top of the solenoid magnet. As a result, it has a large linear position-dependence, a non-negligible non-linear position dependence, a time-dependence, and a longitudinal component parallel to the beam axis at the same time.

To deal with this complexity, we present a following evaluation method. For simplicity we assume that the beam transport by that field can be regarded as linear in the neighborhood of the given reference orbit[2]. Under this assumption, we can evaluate a transfer matrix M by performing a particle tracking simulation. The coordinate of a muon after injection $x_{out} \in (R^4|X, X', Y, Y')$ can be represented by the cordinate before injection x_{in} as $x_{out} = Mx_{in}$.

The VBO amplitude A of each stored muon is written as

$$\begin{aligned} A &= \sqrt{Y_{out}^2 + Y_{out}'^2 R^2/n} = \sqrt{x_{out}^t V x_{out}} \\ &= \sqrt{x_{in}^t W x_{in}} = \sqrt{x_{in}^t P^t \Lambda P x_{in}} \end{aligned} \tag{1}$$

, where

$$V := \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & R^2/n \end{pmatrix}, W := M^t V M,$$

$$P = \begin{pmatrix} v_0 & v_1 & v_2 & v_3 \end{pmatrix}, \Lambda = \begin{pmatrix} \Lambda_0 & 0 & 0 & 0 \\ 0 & \Lambda_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2}$$

by utilizing eigen decomposition of W whose rank is two. In the three-dimensional beam injection, Λ_0, the largest eigen value of W, is much larger than the others because of the strong quadrupole component of magnetic field shown later. In summary, the condition for being inside the storage region is written as a function of x_{in} as follows,

$$\begin{aligned} A < 5\,\text{cm} &\iff \sqrt{x_{in}^t P^t \Lambda P x_{in}} < 5\,\text{cm} \\ &\iff |x_{in} \cdot \mathbf{m}| < 1, \mathbf{m} := v_0\sqrt{\Lambda_0}/5\,\text{cm}. \end{aligned} \tag{3}$$

This is the acceptance characterized by a vector n in the beam phase space.

Acceptance of Three-dimensional Beam Injection

To evaluate the acceptance by Eq. (1)(2)(3), transfer matrix M during the injection is evaluated from the particle tracking simulation as shown in Fig. 2. It is notable that M has a large non-diagonal term of dX'/dY and dY'/dX near the exit of the iron yoke ($z = 110$ cm) due to the skew quadrupole component, dB_X/dX and dB_Y/dY. This is caused by a rapid change of main solenoid field B_z in this region due to the existence of iron yoke and the axial symmetric Maxwell equation of $dB_z/dz + dB_r/dr = 0$.

From the obtained M, the $\mathbf{m}$ is calculated and shown in Fig. 3. The n vector at the injection point is evaluated as $\mathbf{m} = (1/(-60\,\text{m}), 1/(-47\,\text{rad}), 1/142\,\text{m}, 1/84\,\text{rad})$, $1/|\mathbf{m}| = 30$ m or rad.

A subspace denoted by Eq. (3) is the acceptance. As illustrated in Fig. 4, it is geometrically understood as a space between two 3D planes perpendicular to $\mathbf{m}$. It mixes XX' space and YY' space by reflecting the strong XY coupling

[1] It is defined at $z = 160$ cm, a little above the iron yoke top flange, in this paper.

[2] Here we also assume a good knowledge on the magnetic field distribution including its time dependence. This must be validated in the real experiment by a properly designed measuring system.

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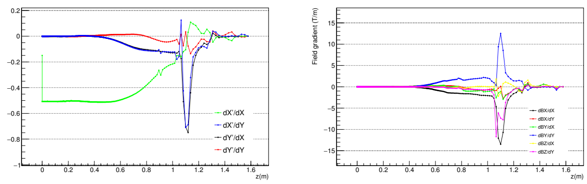

Figure 2: Left: Transfer matrix M calculated every 0.2 ns step along the reference orbit from the injection point (z = 160 cm) to the storage plane (z = 0 cm). Only the interesting one out of 16 components of M are shown. Right: Magnetic field gradient (linear component) around the reference orbit along the orbit. Represented in the beam coordinate.

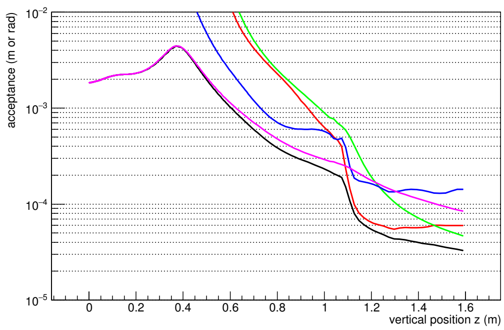

Figure 3: The $\mathbf{m} := (1/\delta X, 1/\delta X', 1/\delta Y, 1/\delta Y')$ vector along the orbit. Black: $1/|\mathbf{m}|$, Red: δX, Green: $\delta X'$, Blue: δY, Magenta: $\delta Y'$.

by the skew quadrupole. The width of the acceptance, the distance between two planes, is $2/|\mathbf{m}|$. To fit the beam distribution to this, we need to apply XY coupling characterized by the direction of $\mathbf{m}$, and to squeeze the distribution width to the level of $1/|\mathbf{m}|$. When a correct XY coupling matching to $\mathbf{m}$ is applied, injection efficiency predicted from this linear beam transport for a beam distribution represented by Sigma matrix Σ is given by $\mathrm{erf}(1/\sqrt{\mathbf{m}^t\Sigma\mathbf{m}}/\sqrt{2})$.

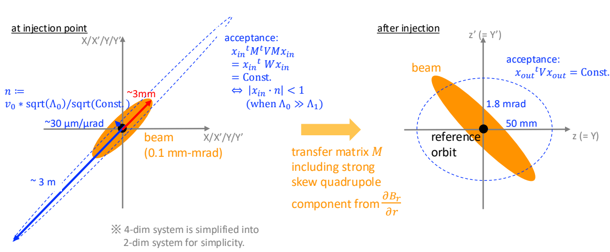

Figure 4: Geometrical illustration of the acceptance.

The small acceptance width can be understood as a feature of the three-dimensional injection and a consequence of basic experiment parameters by a simplified estimation. Pitch angle $\Delta Y'$ after injection must satisfy $|\Delta Y'| < 5\,\mathrm{cm}\sqrt{n}/R$ to be in the storage region. From the equation of motion in a single step under the field gradient, allowed horizontal shift Δr before injection is evaluated as $\Delta Y' = \frac{e\beta c}{m_0\gamma}\Delta t\frac{dB_r}{dr}\Delta r$.

By substituting our experiment parameters[3], we obtain $|\Delta r| = 26\,\mathrm{m}$ and small acceptance width is explained.

Search of Beam Distributions

Beam distributions fitting to the acceptance have been searched. The search was performed by scanning the beam distribution generated from arbitrary Symplectic Matrices and the given beam distribution delivered from the upstream. Fig. 5 shows the result. The injection efficiency, the fraction of muons stored inside the storage region, reaches 80%.

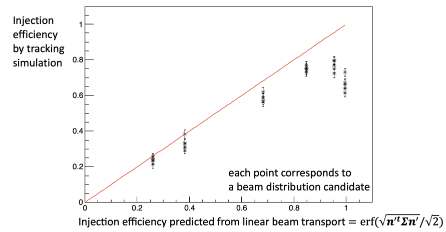

Figure 5: Injection efficiency of searched beam distributions.

Injection efficiency predicted from the particle tracking simulation and that from the above discussion shows a discrepancy. This is interpreted as a non-linearity effect neglected in the acceptance evaluation.

Prospect

Because the injection efficiency seems to be limited by the non-linear effects, inclusion of non-linear effects may be needed for a further improvement of the efficiency.

The evaluated acceptance has a strongly XY coupling and a narrow width. This sets a requirement on the XY coupling of the delivered beam, and also on the stability of the beam center at this precision. To realize three-dimensional beam injection, a careful design of beam diagnosis [6] and tuning system is needed.

Coupling to the ZZ' space is neglected in this paper. Extension to include ZZ' should be straightforward.

Acceptance evaluation in this paper is applicable to other experiments using three-dimensional beam injection into the solenoid magnet and beam extraction from that.

CONCLUSION

In the J-PARC muon g-2/EDM experiment, the muon beam is injected into the compact storage orbit by the three-dimensional beam injection. This paper presents a method to evaluate the acceptance of this scheme. The evaluated acceptance shows a strong XY coupling and a narrow width caused by the skew quadrupole field during injection. Beam distributions fitting to the acceptance have been searched, and injection efficiency of 80% is expected.

[3] $n = 1.5\times10^{-4}, R = 33\,\mathrm{cm}, m_0\gamma = 314\,\mathrm{MeV}, \beta = 0.94, \Delta t = 130\,\mathrm{ns}, dB_r/dr = -dB_z/dz = 3.0\,\mathrm{T}/160\,\mathrm{cm}$

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